$\overrightarrow{NB} = \frac{3}{4}\overrightarrow{AB} - \overrightarrow{AD}$.

8) a) Prin rezolvarea sis ... $\overrightarrow{AB} = \frac{1}{2}\overrightarrow{AC} - \frac{1}{2}\overrightarrow{BD}$ şi

$\overrightarrow{AD} = \frac{1}{2}\overrightarrow{AC} + \frac{1}{2}\overrightarrow{BD}$.

b) $\overrightarrow{AD} = -\overrightarrow{AB} + \overrightarrow{AC}$; $\overrightarrow{BD} =$ ...

9) $\overrightarrow{CD} = -\frac{2}{3}\overrightarrow{BC} = -\frac{2}{3}(\overrightarrow{BA}$ ...

$\overrightarrow{AD} = \frac{2}{3}\overrightarrow{AB} + \frac{1}{3}\overrightarrow{AC}$;

$\overrightarrow{AE} = \frac{4}{5}\overrightarrow{AD} = \frac{8}{15}\overrightarrow{AB} + \frac{4}{15}\overrightarrow{A}$ ...

$\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AE}$ etc.

$\overrightarrow{EC} = \overrightarrow{EA} + \overrightarrow{AC}$ etc.

10) a) $\overrightarrow{AD} = 2\overrightarrow{AB} + 2\overrightarrow{AF}$; ... ;

d) $\overrightarrow{AD} = \frac{2}{3}\overrightarrow{AC} + \frac{2}{3}\overrightarrow{AE}$.

11) a) $\overrightarrow{AC} = -\overrightarrow{BA} + \overrightarrow{BC}$; ... $\overrightarrow{BC} = -\frac{1}{3}\overrightarrow{BA} - \frac{1}{3}\overrightarrow{BC} +$

$\overrightarrow{BC} = -\frac{1}{3}\overrightarrow{BA} + \frac{2}{3}\overrightarrow{BC}$; **d)** A ... $-\frac{2}{3}\overrightarrow{BA} + \frac{1}{3}\overrightarrow{BC}$;

e) $\overrightarrow{GM} = \overrightarrow{GB} + \overrightarrow{BM} = -\frac{1}{3}$ ...

12) a) $\overrightarrow{AM} = \frac{2}{3}\overrightarrow{AB} + \frac{1}{3}\overrightarrow{AC}$; ... ; $\overrightarrow{AN} = \frac{1}{2}\overrightarrow{AM} + \frac{1}{2}\overrightarrow{AC}$.

13) a) $\overrightarrow{AB} = 3\overrightarrow{AM} + 0 \cdot$ ... $3\overrightarrow{AN}$;

d) $\overrightarrow{BD} =$ ...

14) a) $\overrightarrow{CD} = -k$ ...

c) Dacă E ∈ (AB) şi D ... AE = AB − DC.

$\overrightarrow{BC} = \overrightarrow{ED} = \overrightarrow{EA} + \overrightarrow{AD} = -$ ...

d) $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} = k\overrightarrow{AB} + \overrightarrow{AD}$; **e)** ... $\overrightarrow{AD}$;

f) $\frac{PD}{PA} = k \Rightarrow \frac{PD}{AD} = \frac{k}{1-k} \Rightarrow \overrightarrow{PD} = \frac{k}{1-k}\overrightarrow{DA} =$ ... $\cdot \overrightarrow{AB}$.

Vectori coliniari. Probleme de coliniaritate

1) a) $\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BC}$, $\overrightarrow{PN} = \frac{1}{2}\overrightarrow{BC}$, $\overrightarrow{AB} + \overrightarrow{CA} = -\overrightarrow{BC}$, $\overrightarrow{PA} + \overrightarrow{PM} = \overrightarrow{PN} = \frac{1}{2}\overrightarrow{BC}$.

b) $\overrightarrow{AG} = \frac{2}{3}\overrightarrow{AM}$, $\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AM}$, $\overrightarrow{MP} + \overrightarrow{MN} = \overrightarrow{MA} = -\overrightarrow{AM}$, $\overrightarrow{GM} = -\frac{1}{3}$AM.

2) Dacă $\vec{b} = \vec{0}$, atunci $\vec{v} = 2\vec{a} = \frac{3}{2}\vec{u}$.

Dacă $\vec{b} \neq \vec{0}$, atunci există $\alpha \in \mathbb{R}$ astfel încât $\vec{a} = \alpha\vec{b}$, deci $\vec{u} = (3\alpha - 2)\vec{b}$ şi $\vec{v} = (2\alpha - 3)\vec{b}$.

3) Dacă $\vec{v} = \vec{0}$, atunci $\vec{b} = 2\vec{a}$.

Dacă $\vec{v} \neq \vec{0}$, atunci există $\alpha \in \mathbb{R}$ astfel încât $\vec{u} = \alpha\vec{v}$, adică $\vec{a} + 2\vec{b} = 2\alpha\vec{a} - \alpha\vec{b}$, de unde $(2\alpha - 1)\vec{a} = (2 + \alpha)\vec{b}$.

4) $\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ} = -\frac{4}{5}\overrightarrow{AB} + \frac{1}{3}\overrightarrow{AC}$.

$\overrightarrow{RP} = \overrightarrow{RB} + \overrightarrow{BP} = \frac{1}{7}\overrightarrow{BC} - \frac{1}{5}\overrightarrow{AB} = \frac{1}{7}\left(\overrightarrow{BA} + \overrightarrow{AC}\right) - \frac{1}{5}\overrightarrow{AB} = -\frac{12}{35}\overrightarrow{AB} + \frac{1}{7}\overrightarrow{AC} = \frac{3}{7}\overrightarrow{PQ}$.

5) $\overrightarrow{CD} = \overrightarrow{CN} + \overrightarrow{ND} = 3\overrightarrow{NP} + \overrightarrow{MN}$.

$\overrightarrow{CE} = \overrightarrow{CP} + \overrightarrow{PE} = 4\overrightarrow{NP} + 2\overrightarrow{MP} = 4\overrightarrow{NP} + 2\left(\overrightarrow{MN} + \overrightarrow{NP}\right) = 6\overrightarrow{NP} + 2\overrightarrow{MN} = 2\overrightarrow{CD}$.

6) $\overrightarrow{BP} = \overrightarrow{BA} + \overrightarrow{AP} = -\overrightarrow{AB} + \overrightarrow{AM} + \overrightarrow{AN} = -\overrightarrow{AB} + \frac{3}{4}\overrightarrow{AB} + \frac{3}{4}\overrightarrow{AC} = -\frac{1}{4}\overrightarrow{AB} + \frac{3}{4}\overrightarrow{AC}$.

$\overrightarrow{PC} = \overrightarrow{PB} + \overrightarrow{BC} = \frac{1}{4}\overrightarrow{AB} - \frac{3}{4}\overrightarrow{AC} - \overrightarrow{AB} + \overrightarrow{AC} = -\frac{3}{4}\overrightarrow{AB} + \frac{1}{4}\overrightarrow{AC}$.

7) a) $\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AE} = -\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AD}$;

$\overrightarrow{RS} = \overrightarrow{RC} + \overrightarrow{CS} = \frac{2}{3}\overrightarrow{AB} - \frac{1}{3}\overrightarrow{AD}$.

b) $\overrightarrow{RS} = -\frac{2}{3}\text{BC}$.

8) a) $\overrightarrow{MN} = -\frac{1}{3}\overrightarrow{DM} = -\frac{1}{3}\left(\overrightarrow{DA} + \overrightarrow{AM}\right) = \frac{1}{3}\overrightarrow{AD} - \frac{1}{6}\overrightarrow{AB}$;

b) $\overrightarrow{AN} = \overrightarrow{AM} + \overrightarrow{MN} = \frac{1}{2}\overrightarrow{AB} + \frac{1}{3}\overrightarrow{AD} - \frac{1}{6}\overrightarrow{AB} = \frac{1}{3}\left(\overrightarrow{AB} + \overrightarrow{AD}\right) = \frac{1}{3}\overrightarrow{AC}$.

9) AB ∥ CD ⇒ $\exists k \in (0;\ \infty)$ astfel încât $\overrightarrow{AB} = k\overrightarrow{DC}$.

$\overrightarrow{MA} = \frac{1}{2}\overrightarrow{CA} = \frac{1}{2}\left(\overrightarrow{CD} + \overrightarrow{DA}\right) = \frac{1}{2}\left(-\overrightarrow{DC} + \overrightarrow{DA}\right)$;

$\overrightarrow{BN} = \frac{1}{2}\overrightarrow{BD} = \frac{1}{2}\left(\overrightarrow{BA} + \overrightarrow{AD}\right) = \frac{1}{2}\left(-k\overrightarrow{DC} - \overrightarrow{DA}\right)$;

$\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AB} + \overrightarrow{BN} = -\frac{1}{2}\overrightarrow{DC} + \frac{1}{2}\overrightarrow{DA} - \frac{k}{2}\overrightarrow{DC} - \frac{1}{2}\overrightarrow{DA} + k\overrightarrow{DC} = \frac{k-1}{2}\overrightarrow{DC} = \frac{k-1}{2k}\overrightarrow{AB}$.

10) $\overrightarrow{BE} = \frac{1}{2}\left(\overrightarrow{BA} + \overrightarrow{BD}\right) = \frac{1}{2}\overrightarrow{BA} + \frac{1}{4}\overrightarrow{BC}$;

$\overrightarrow{BF} = \frac{2}{3}\overrightarrow{BA} + \frac{1}{3}\overrightarrow{BC} = \frac{4}{3}\left(\frac{1}{2}\overrightarrow{BA} + \frac{1}{4}\overrightarrow{BC}\right)$;

$\frac{\overrightarrow{BE}}{\overrightarrow{BF}} = \frac{3}{4}$.

11) Dacă se notează $\frac{\text{BE}}{\text{BA}} = k$, atunci $\frac{\text{EF}}{\text{AC}} = k$ şi $\frac{\text{EG}}{\text{AD}} = k$.

$\overrightarrow{FG} = \overrightarrow{FE} + \overrightarrow{EG} = k\overrightarrow{CA} + k\overrightarrow{AD} = k\overrightarrow{CD}$.

12) „⟹" Considerăm vectorii $\vec{u}$ şi $\vec{v}$ coliniari. Dacă $\vec{v} = \vec{0}$, atunci $\vec{u} + \vec{v} = \vec{u} - \vec{v}$. Dacă $\vec{v} \neq \vec{0}$, atunci există $k \in \mathbb{R}$ astfel încât $\vec{u} = k\vec{v}$.

$\vec{u} + \vec{v} = (1 + k)\vec{u}$ şi $\vec{u} - \vec{v} = (1 + k)\vec{u}$, deci $\vec{u} + \vec{v}$ şi $\vec{u} - \vec{v}$ sunt coliniari.

„⟸"Considerăm vectorii $\vec{u} + \vec{v}$ şi $\vec{u} - \vec{v}$ coliniari. Dacă $\vec{u} - \vec{v} = \vec{0}$, atunci $\vec{u} = \vec{v}$. Dacă $\vec{u} - \vec{v} \neq \vec{0}$, atunci există $k \in \mathbb{R}$ astfel încât $\vec{u} + \vec{v} = k(\vec{u} - \vec{v})$, de unde $(k - 1)\vec{u} = (1 + k)\vec{v}$, deci $\vec{u}$ şi $\vec{v}$ sunt coliniari.

13) (p1) este falsă:

Se consideră dreptunghiul ABCD şi se notează $\overrightarrow{AB} = \vec{a}$ şi $\overrightarrow{AD} = \vec{b}$. Atunci $\overrightarrow{AC} = \vec{a} + \vec{b}$ şi $\overrightarrow{DB} = \vec{a} - \vec{b}$, deci $\left|\overrightarrow{AC}\right| = \left|\overrightarrow{DB}\right|$ şi vectorii $\vec{a}$ şi $\vec{b}$ nu sunt coliniari.

(p2) este adevărată:

Presupunem că $\vec{a}$ şi $\vec{b}$ sunt necoliniari şi considerăm triunghiul ABC astfel încât $\overrightarrow{AB} = \vec{a}$ şi $\overrightarrow{BC} = \vec{b}$. Rezultă că $|\overrightarrow{AC}| = |\overrightarrow{AB} + \overrightarrow{BC}| = |\overrightarrow{AB}| + |\overrightarrow{BC}|$, contradicție.

(p3) este falsă:

Dacă $\vec{b} = -\vec{a}, \vec{b} \neq \vec{0}$, atunci $|\vec{a} + \vec{b}| = 0 \neq |\vec{a}| + |\vec{b}|$.

14) AMCQ paralelogram $\Rightarrow$ EH ∥ FG;

BNDP paralelogram $\Rightarrow$ EF ∥ GH;

$\Delta DEQ = \Delta NEA \Rightarrow [DE] \equiv [EN]$.

$\overrightarrow{AE} = \frac{1}{2}(\overrightarrow{AD} + \overrightarrow{AN}) = \frac{1}{2}\overrightarrow{AD} + \frac{3}{8}\overrightarrow{AB}$.

$\Delta BGM \equiv \Delta PGC \Rightarrow [MG] \equiv [CG]$.

$\overrightarrow{BG} = \frac{1}{2}(\overrightarrow{BM} + \overrightarrow{BC}) = \frac{1}{2}\overrightarrow{AD} - \frac{3}{8}\overrightarrow{AB}$.

$\overrightarrow{EG} = \overrightarrow{EA} + \overrightarrow{AB} + \overrightarrow{BG} = \frac{1}{4}\overrightarrow{AB} \Rightarrow EG \parallel AB$.

15) $\Delta DEP \sim \Delta MEA \Rightarrow \frac{DE}{EM} = \frac{DP}{AM} = \frac{5}{3} \Rightarrow \overrightarrow{DE} = \frac{5}{3}\overrightarrow{EM} \Rightarrow \overrightarrow{AE} = \frac{3}{8}\overrightarrow{AD} + \frac{5}{8}\overrightarrow{AM} = \frac{3}{8}\overrightarrow{AD} + \frac{1}{8}\overrightarrow{AB}$.

$\Delta DFQ \sim \Delta NFA \Rightarrow \frac{DF}{FN} = \frac{DQ}{AN} = \frac{6}{5} \Rightarrow \overrightarrow{DF} = \frac{6}{5}\overrightarrow{FN} \Rightarrow \overrightarrow{AF} = \frac{5}{11}\overrightarrow{AD} + \frac{6}{11}\overrightarrow{AN} = \frac{5}{11}\overrightarrow{AD} + \frac{4}{11}\overrightarrow{AB}$.

$\overrightarrow{EF} = \overrightarrow{EA} + \overrightarrow{AF} = \frac{21}{88}\overrightarrow{AB} + \frac{7}{88}\overrightarrow{AD} = \frac{7}{88}(3\overrightarrow{AB} + \overrightarrow{AD})$. (1)

$\Delta GBN \sim \Delta GQC \Rightarrow \frac{GB}{GQ} = \frac{5}{3} \Rightarrow \overrightarrow{BG} = \frac{5}{3}\overrightarrow{GQ} \Rightarrow \overrightarrow{AG} = \frac{3}{8}\overrightarrow{AB} + \frac{5}{8}\overrightarrow{AQ} = \frac{3}{8}\overrightarrow{AB} + \frac{5}{8}\left(\overrightarrow{AD} + \frac{4}{5}\overrightarrow{AB}\right) =$

$= \frac{7}{8}\overrightarrow{AB} + \frac{5}{8}\overrightarrow{AD}$.

$\overrightarrow{GF} = \overrightarrow{GA} + \overrightarrow{AF} = -\frac{7}{8}\overrightarrow{AB} - \frac{5}{8}\overrightarrow{AD} + \frac{5}{11}\overrightarrow{AD} + \frac{4}{11}\overrightarrow{AB} = -\frac{45}{88}\overrightarrow{AB} - \frac{15}{88}\overrightarrow{AD} =$

$= -\frac{15}{88}(3\overrightarrow{AB} + \overrightarrow{AD})$. (2)

Din (1) şi (2) rezultă că vectorii $\overrightarrow{EF}$ şi $\overrightarrow{GF}$ sunt coliniari.

16) Dacă $\frac{DC}{AB} = k$, atunci $\overrightarrow{DC} = k\overrightarrow{AB}$.

$\frac{DE}{EN} = \frac{DP}{AN} = \frac{\frac{2}{3}DC}{\frac{2}{3}AC} = k \Rightarrow \overrightarrow{DE} = k\overrightarrow{EN}$.

$\overrightarrow{AE} = \frac{1}{1+k}\overrightarrow{AD} + \frac{k}{1+k}\overrightarrow{AN} = \frac{1}{1+k}\overrightarrow{AD} + \frac{2k}{3(1+k)}\overrightarrow{AB}$.

$\frac{CF}{MF} = \frac{DC}{AB} = k \Rightarrow \overrightarrow{CF} = k\overrightarrow{MF}$.

$\overrightarrow{BF} = \frac{1}{1+k}\overrightarrow{BC} + \frac{k}{1+k}\overrightarrow{BM} = \frac{1}{1+k}(\overrightarrow{BA} + \overrightarrow{AD} + k\overrightarrow{AB}) - \frac{2k}{3(1+k)}\overrightarrow{AB} = \frac{k-3}{3(1+k)}\overrightarrow{AB} + \frac{1}{1+k}\overrightarrow{AD}$.

$\overrightarrow{EF} = \overrightarrow{EA} + \overrightarrow{AB} + \overrightarrow{BF} = \frac{2k}{3(1+k)}\overrightarrow{AB} \Rightarrow EF \parallel AB$.

17) $\overrightarrow{BM} = 3\overrightarrow{MF} \Rightarrow \overrightarrow{AM} = \frac{1}{4}\overrightarrow{AB} + \frac{3}{4}\overrightarrow{AF}$.

$\overrightarrow{AB} + \overrightarrow{AM} = \frac{5}{4}\overrightarrow{AB} + \frac{3}{4}\overrightarrow{AF}$.

Dacă $AB \cap BF = \{P\}$ atunci $\overrightarrow{MN} = \overrightarrow{MP} + \overrightarrow{PN} = -\frac{1}{4}\overrightarrow{BF} + \frac{1}{2}\overrightarrow{AD} = \frac{1}{4}\overrightarrow{AB} - \frac{1}{4}\overrightarrow{AF} + \overrightarrow{AB} + \overrightarrow{AF} = \frac{5}{4}\overrightarrow{AB} + \frac{3}{4}\overrightarrow{AF}$.