# FUNCȚII TRIGONOMETRICE

**1) a)** $(0;\pi)\cup(\pi;2\pi)$; **b)** ; $\left[0;\frac{\pi}{3}\right)\cup\left(\frac{\pi}{3};\frac{5\pi}{3}\right)\cup\left(\frac{5\pi}{3};2\pi\right)$ **c)** $[0;\pi]$; **d)** $\left[0;\frac{\pi}{2}\right]\cup\left[\frac{3\pi}{2};2\pi\right)$;
**e)** $\left(\frac{\pi}{6};\frac{5\pi}{6}\right)$; **f)** $[0;2\pi)\setminus\left\{\frac{2\pi}{3};\pi;\frac{4\pi}{3}\right\}$; **g)** $[0;2\pi)\setminus\left\{\frac{\pi}{4};\frac{3\pi}{4};\frac{5\pi}{4};\frac{7\pi}{4}\right\}$; **h)** $[0;2\pi)\setminus\left\{\frac{\pi}{6};\frac{5\pi}{6};\frac{3\pi}{2}\right\}$;
**i)** $(0;2\pi)\setminus\left\{\frac{\pi}{2};\pi;\frac{3\pi}{2}\right\}$; **j)** $(0;2\pi)\setminus\left\{\frac{2\pi}{3};\pi;\frac{4\pi}{3};\frac{5\pi}{3};\frac{\pi}{3}\right\}$; **k)** $\left(0;\frac{\pi}{2}\right]\cup\left(\pi;\frac{3\pi}{2}\right]$;
**l)** $\left[\frac{\pi}{4};\frac{\pi}{2}\right)\cup\left[\frac{5\pi}{4};\frac{3\pi}{2}\right)$; **m)** $[0;2\pi)\setminus\left\{\frac{\pi}{4};\frac{\pi}{2};\frac{3\pi}{4};\frac{3\pi}{2};\frac{5\pi}{4};\frac{7\pi}{4}\right\}$; **n)** $[0;2\pi)\setminus\left\{\frac{\pi}{6};\frac{5\pi}{6};\frac{7\pi}{6};\frac{11\pi}{6};\pi\right\}$.
**2) a)** $A\left(\frac{\pi}{2};0\right)$, $B(0;1)$; **b)** $A(0;3)$; **c)** $A(0;2)$, $B\left(\frac{\pi}{2};0\right)$, $C\left(\frac{3\pi}{2};0\right)$; **d)** $A_k\left(\frac{k\pi}{5};0\right)$, $k\in\mathbb{Z}$;
**e)** $A(0;-1)$, $B\left(\frac{\pi}{4};0\right)$, $C\left(\frac{5\pi}{4};0\right)$; **f)** $A\left(\frac{\pi}{3};0\right)$, $B\left(\frac{2\pi}{3};0\right)$; **g)** $O(0;0)$; **h)** $A(0;\log_5 2)$.
**3)** Funcțiile de la a), d) și f) sunt impare; funcțiile de la b) și e) sunt pare; funcțiile de la c) și g) nu sunt nici pare, nici impare.
**4)** Se demonstrează că $f(x+T)=f(x)$, $\forall x\in D$.
**5) a)** $T\in\mathbb{R}^*$ este perioadă pentru $f \Leftrightarrow \sin\frac{7(x+T)}{2}=\sin\frac{7x}{2}$, $\forall x\in\mathbb{R}\Leftrightarrow$
$\Leftrightarrow \sin\frac{7x+7T}{2}-\sin\frac{7x}{2}=0$, $\forall x\in\mathbb{R}\Leftrightarrow 2\sin\frac{7T}{4}\cos\frac{14x+7T}{4}=0$, $\forall x\in\mathbb{R}\Leftrightarrow\sin\frac{7T}{4}=0\Leftrightarrow$
$\Leftrightarrow\frac{7T}{4}=k\pi$, $k\in\mathbb{Z}^*\Leftrightarrow T=\frac{4k\pi}{7}$, $k\in\mathbb{Z}^*$.
**b)** $3k\pi$, $k\in\mathbb{Z}^*$; **c)** $\frac{k\pi}{2}$, $k\in\mathbb{Z}^*$; **d)** $\frac{2k\pi}{3}$, $k\in\mathbb{Z}^*$.
**6) a)** $\min f=-5$, $\max f=5$; **b)** $\min f=-4$, $\max f=-2$; **c)** $\min f=0$, $\max f=1$;
**d)** $\min f=0$, $\max f=1$; **e)** $\min f=-\sqrt{3}$, $\max f=1$; **f)** $\min f=-\sqrt{3}$, $\max f=0$;
**g)** $\min f=1$, $\max f=5$; **h)** $\min f=2-\sqrt{3}$, $\max f=3$;
**i)** $\min f=\frac{3+\sqrt{3}}{3}$, $\max f=1+\sqrt{3}$.
**7) a)** $f$ este strict crescătoare pe $\left[0;\frac{\pi}{2}\right]$ și strict descrescătoare pe $\left[\frac{\pi}{2};\pi\right]$.
**b)** $f$ este strict crescătoare pe $\left[-\frac{\pi}{2};0\right]$ și strict descrescătoare pe $\left[0;\frac{\pi}{2}\right]$.
**c)** $f$ este strict crescătoare;
**d)** $f$ este strict descrescătoare;
**e)** $f$ este strict descrescătoare pe $\left[\frac{\pi}{2};\frac{3\pi}{2}\right]$ și strict crescătoare pe $\left[\frac{3\pi}{2};2\pi\right]$.
**f)** $f$ este strict crescătoare pe $(0;\pi]$ și strict descrescătoare pe $\left[\pi;\frac{3\pi}{2}\right)$.
**g)** $f$ este strict crescătoare;
**h)** $f$ este strict descrescătoare;
**i)** $f$ este strict crescătoare;
**j)** $f$ este strict crescătoare;
**k)** $f$ este strict crescătoare.
**8) a)** $a\in\left[\frac{\pi}{3};\frac{\pi}{2}\right]$; **b)** $a\in\left[\frac{\pi}{2};\pi\right]$; **c)** $a\in\left[\frac{\pi}{2};\frac{5\pi}{4}\right)$; **d)** ; $a\in\left[\frac{5\pi}{4};2\pi\right)$.
**9) a)** $f(x)=\begin{cases}\cos x-\sin x, x\in\left[0;\frac{\pi}{4}\right)\cup\left[\frac{5\pi}{4};2\pi\right)\\ \sin x-\cos x, x\in\left(\frac{\pi}{4};\frac{5\pi}{4}\right)\end{cases}$;
**b)** $f(x)=\begin{cases}\text{tg}\,x-\text{ctg}\,x, x\in\left[\frac{\pi}{4};\frac{\pi}{2}\right)\\ \text{ctg}\,x-\text{tg}\,x, x\in\left(0;\frac{\pi}{4}\right)\end{cases}$

**c)** $f(x)=\begin{cases}1, x\in\left\{0;\frac{\pi}{2}\right\}\\ 0, x\in\left(0;\frac{\pi}{2}\right)\\ -1, x\in\left(\frac{\pi}{2};\pi\right]\cup\left[\frac{3\pi}{2};2\pi\right)\\ -2, x\in\left(0;\frac{3\pi}{2}\right)\end{cases}$

**d)** $f(x)=\begin{cases}\sin x, x\in[0;\pi]\setminus\left\{\frac{\pi}{2}\right\}\\ \sin x+1, x\in(\pi;2\pi)\setminus\left\{\frac{3\pi}{2}\right\}\\ 0, x\in\left\{\frac{\pi}{2};\frac{3\pi}{2}\right\}\end{cases}$

**e)** $f(x)=\begin{cases}\operatorname{ctg} x, x\in\left(\frac{\pi}{2};\frac{3\pi}{4}\right]\\ \operatorname{tg} x, x\in\left(\frac{3\pi}{4};\pi\right)\end{cases}$

**f)** $f(x)=\begin{cases}\cos x, x\in\left[\frac{\pi}{2};\frac{5\pi}{4}\right]\\ \sin x, x\in\left(\frac{5\pi}{4};2\pi\right]\end{cases}$.

**10) a)** $[0;1]$; **b)** $\left[-\frac{\sqrt{2}}{2};1\right]$; **c)** $[-1;1]$; **d)** $[-1;0)$; **e)** $\left[-\frac{\sqrt{3}}{2};1\right]$; **f)** $[-1;0]$; **g)** $(-1;1)$.

**11) a)** $\mathbb{R}$; **b)** $\mathbb{R}$; **c)** $\left[-1;\sqrt{3}\right]$; **d)** $\left[-\sqrt{3};-1\right]$; **e)** $\left(-\sqrt{3};1\right)$; **f)** $\mathbb{R}^*$; **g)** $\left[-1;-\frac{\sqrt{3}}{3}\right]\cup\left[\frac{\sqrt{3}}{3};\sqrt{3}\right]$.

**12) a)** $[0;2\pi)$; **b)** $[0;\pi]$; **c)** $[\pi;2\pi)\cup\{0\}$; **d)** $\left\{\frac{\pi}{4};\frac{3\pi}{4};\frac{5\pi}{4};\frac{7\pi}{4}\right\}$; **e)** $\left[\frac{\pi}{6};\frac{5\pi}{6}\right]$;
**f)** $\left(\pi;\frac{7\pi}{6}\right)\cup\left(\frac{11\pi}{6};2\pi\right)$; **g)** $\left[0;\frac{\pi}{3}\right)\cup\left(\frac{2\pi}{3};\frac{5\pi}{3}\right)$;

**13) a)** $[0;2\pi)$; **b)** $\left(0;\frac{\pi}{2}\right)\cup\left(\frac{3\pi}{2};2\pi\right)$; **c)** $\left[\frac{\pi}{2};\frac{3\pi}{2}\right]$; **d)** $\left\{\frac{\pi}{6};\frac{2\pi}{3};\frac{4\pi}{3};\frac{11\pi}{6}\right\}$; **e)** $\left[0;\frac{3\pi}{4}\right]\cup\left[\frac{5\pi}{4};2\pi\right]$;
**f)** $\left(\frac{\pi}{3};\frac{\pi}{2}\right)\cup\left(\frac{3\pi}{2};\frac{5\pi}{3}\right)$; **g)** $\left(\frac{\pi}{6};\frac{5\pi}{6}\right)\cup\left(\frac{7\pi}{6};\frac{11\pi}{6}\right)$.

**14) a)** $(0;2\pi)\setminus\left\{\frac{\pi}{2};\frac{3\pi}{2}\right\}$; **b)** $\left(0;\frac{\pi}{2}\right)\cup\left[\pi;\frac{3\pi}{2}\right)$; **c)** $\left(\frac{\pi}{2};\pi\right)\cup\left(\frac{3\pi}{2};2\pi\right)$; **d)** $\left(0;\frac{\pi}{4}\right]\cup\left[\pi;\frac{5\pi}{4}\right]$;
**e)** $\left(\frac{\pi}{2};\frac{2\pi}{3}\right)\cup\left(\frac{3\pi}{2};\frac{5\pi}{3}\right)$; **f)** $\left\{\frac{\pi}{4};\frac{3\pi}{4};\frac{5\pi}{4};\frac{7\pi}{4}\right\}$; **g)** $\left[\frac{\pi}{4};\frac{\pi}{2}\right)\cup\left[\frac{5\pi}{4};\frac{3\pi}{2}\right)$;
**h)** $\left(0;\frac{\pi}{3}\right]\cup\left[\frac{5\pi}{6};\frac{4\pi}{3}\right]\cup\left[\frac{11\pi}{6};2\pi\right)$.

**15) a)** $(0;\pi)\cup(\pi;2\pi)$; **b)** $\left(0;\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2};\pi\right)\cup\left(\pi;\frac{3\pi}{2}\right)\cup\left(\frac{3\pi}{2};2\pi\right)$; **c)** $\left(\frac{\pi}{2};\pi\right)\cup\left(\frac{3\pi}{2};2\pi\right)$;
**d)** $\left[\frac{\pi}{4};\frac{3\pi}{4}\right]\cup\left[\frac{5\pi}{4};\frac{7\pi}{4}\right]$; **e)** $\left(0;\frac{\pi}{6}\right]\cup\left(\pi;\frac{7\pi}{6}\right]$; **f)** $\left\{\frac{\pi}{6};\frac{\pi}{4};\frac{7\pi}{6};\frac{5\pi}{4}\right\}$; **g)** $\left(\frac{3\pi}{4};\frac{5\pi}{6}\right)\cup\left(\frac{7\pi}{4};\frac{11\pi}{6}\right)$;
**h)** $\left(0;\frac{\pi}{4}\right)\cup\left(\frac{3\pi}{4};\pi\right)\cup\left(\pi;\frac{5\pi}{4}\right)\cup\left(\frac{7\pi}{4};2\pi\right)$.

**16) a)** $\frac{7\pi}{6}$; **b)** $-\frac{\pi}{6}$; **c)** $\frac{7\pi}{12}$; **d)** $\frac{5(\pi)^2}{36}$; **e)** $\frac{3\pi}{2}$; **f)** $\frac{2\pi}{3}$; **g)** $\frac{\pi}{3}$; **h)** $\frac{3\pi}{2}$.

**17) a)** $\left[-\frac{1}{3};\frac{1}{3}\right]$; **b)** $[1;2]$; **c)** $[-1;1]$; **d)** $(-\infty;-1]\cup[0;\infty)$; **e)** $[-3;3]$;
**f)** $(-\infty;-2]\cup[2;\infty)$; **g)** $\left(-\infty;\frac{1}{3}\right]\cup[5;\infty)$; **h)** $[-1;1]$; **i)** $\mathbb{R}\setminus\{-4;1\}$; **j)** $\left[\frac{1}{2};\infty\right)$;
**k)** $\mathbb{R}\setminus\left\{\pm\sqrt{7}\right\}$; **l)** $\mathbb{R}\setminus\left\{-\frac{1}{3}\right\}$; **m)** $(-\infty;-1]\cup(2;\infty)$; **n)** $\mathbb{R}\setminus\left\{\frac{1}{2};2\right\}$.

**18) a)** $O(0;0)$; **b)** $A\left(0;\frac{\pi}{2}\right)$, $B(-1;0)$; **c)** $A\left(0;\frac{\pi}{2}\right)$, $B\left(\frac{1}{3};0\right)$; **d)** $A(1;0)$; **e)** $O(0;0)$;
**f)** $A\left(0;\frac{\pi}{4}\right)$, $B\left(\frac{1}{3};0\right)$; **g)** $A\left(0;\frac{\pi}{2}\right)$; **h)** $A\left(0;\frac{5\pi}{4}\right)$.

**19) a)** $\frac{\pi}{5}$; **b)** $-\frac{\pi}{7}$; **c)** $\frac{\pi}{9}$; **d)** $\frac{\pi}{5}$; **e)** $0$; **f)** $\frac{\pi}{6}$; **g)** $\pi$; **h)** $\frac{2\pi}{3}$;
**i)** $3\in\left(\frac{\pi}{2};\pi\right)\Rightarrow\pi-3\in\left(0;\frac{\pi}{2}\right)$.
$\arcsin(\sin 3)=\arcsin(\sin(\pi-3))=\pi-3$.

**j)** $7 \in \left(2\pi; \frac{5\pi}{2}\right) \Rightarrow 2\pi - 7 \in \left(-\frac{\pi}{2}; 0\right)$

$\arcsin(\sin(-7)) = \arcsin(\sin(2\pi - 7)) = 2\pi - 7.$

**k)** $\pi - \frac{5}{2}$; **l)** $2\pi - 4$; **m)** $\frac{3}{2}$;

**n)** $11 \in \left(\frac{7\pi}{2}; 4\pi\right) \Rightarrow 4\pi - 11 \in \left(0; \frac{\pi}{2}\right).$

$\arccos(\cos 11) = \arccos(\cos(4\pi - 11)) = 4\pi - 11.$

**20) a)** $\frac{\pi}{9}$; **b)** $- -\frac{2\pi}{7}$; **c)** $\frac{2\pi}{5}$;

**d)** $\operatorname{arcctg}\left(\operatorname{ctg}\left(-\frac{\pi}{7}\right)\right) = \operatorname{arcctg}\left(-\operatorname{ctg}\frac{\pi}{7}\right) = \pi - \operatorname{arcctg}\left(\operatorname{ctg}\frac{\pi}{7}\right) = \pi - \frac{\pi}{7} = \frac{6\pi}{7}.$

**e)** $0$; **f)** $\frac{\pi}{3}$; **g)** $\frac{\pi}{4}$; **h)** $\frac{\pi}{4}$;

**i)** $\frac{7}{2} \in \left(\pi; \frac{3\pi}{2}\right) \Rightarrow \pi - \frac{7}{2} \in \left(-\frac{\pi}{2}; 0\right).$

$\operatorname{arctg}\left(\operatorname{tg}\frac{7}{2}\right) = \operatorname{arctg}\left(-\operatorname{tg}\left(\pi - \frac{7}{2}\right)\right) = -\operatorname{arctg}\left(\operatorname{tg}\left(\pi - \frac{7}{2}\right)\right) = -\left(\pi - \frac{7}{2}\right) = \frac{7}{2} - \pi.$

**j)** $\pi - 3$; **k)** $7 - 2\pi$; **l)** $\pi - 3$.

**21) a)** $\frac{1}{5}$; **b)** $-\frac{2}{3}$; **c)** $\frac{2}{7}$; **d)** $-\frac{1}{3}$; **e)** $\frac{3}{5}$; **f)** $\frac{12}{13}$;

**g)** $\sin\left(2\arcsin\frac{3}{5}\right) = 2\sin\left(\arcsin\frac{3}{5}\right)\cos\left(\arcsin\frac{3}{5}\right) = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}.$

**h)** $\cos\left(2\arcsin\frac{3}{5}\right) = 1 - 2\sin^2\left(\arcsin\frac{3}{5}\right) = \frac{7}{25}.$

**i)** $\cos\left(2\arccos\frac{3}{5}\right) = 2\cos^2\left(\arccos\frac{3}{5}\right) - 1 = -\frac{7}{25}.$

**j)** $\operatorname{tg}\left(\arcsin\frac{\sqrt{3}}{3}\right) = \frac{\sin\left(\arcsin\frac{\sqrt{3}}{3}\right)}{\cos\left(\arcsin\frac{\sqrt{3}}{3}\right)} = \frac{\sqrt{2}}{2}.$

**k)** $-2\sqrt{2}$; **l)** $-\frac{2\sqrt{21}}{21}.$

**22) a)** $7$; **b)** $-3$; **c)** $\sqrt{5}$; **d)** $-2$; **e)** $\frac{1}{4}$; **f)** $-\frac{1}{3}$;

**g)** $\operatorname{tg}\left(2\operatorname{arctg}\frac{1}{3}\right) = \frac{2\operatorname{tg}\left(\operatorname{arctg}\frac{1}{3}\right)}{1 - \operatorname{tg}^2\left(\operatorname{arctg}\frac{1}{3}\right)} = \frac{3}{4};$

**h)** $\operatorname{ctg}\left(\frac{1}{2}\arcsin\frac{4}{5}\right) = \frac{1 + \cos\left(\arcsin\frac{4}{5}\right)}{\sin\left(\arcsin\frac{4}{5}\right)} = 2;$

**i)** $\sin(\operatorname{arctg} 2) = \frac{\operatorname{tg}(\operatorname{arctg} 2)}{\sqrt{1 + \operatorname{tg}^2(\operatorname{arctg} 2)}} = \frac{2\sqrt{5}}{5};$

**j)** $\cos(\operatorname{arctg}(-3)) = \frac{1}{\sqrt{1 + \operatorname{tg}^2(\operatorname{arctg}(-3))}} = \frac{\sqrt{10}}{10}.$

**23) a)** $\sin\left(\arcsin\frac{1}{3} + \arccos\frac{2}{3}\right) = \sin\left(\arcsin\frac{1}{3}\right) \cdot \cos\left(\arccos\frac{2}{3}\right) + \cos\left(\arcsin\frac{1}{3}\right) \cdot$

$\cdot \sin\left(\arccos\frac{2}{3}\right) = \frac{2}{9} + \sqrt{1 - \frac{1}{9}} \cdot \sqrt{1 - \frac{4}{9}} = \frac{2\left(1 + 2\sqrt{10}\right)}{9}$

**b)** $\frac{\sqrt{6}}{9}$; **c)** $-\frac{4}{7}$; **d)** $-7$;

**e)** $\operatorname{ctg}(\operatorname{arctg} 7 + \operatorname{arcctg} 2) = \frac{\operatorname{ctg}(\operatorname{arctg} 7)\operatorname{ctg}(\operatorname{arcctg} 2) - 1}{\operatorname{ctg}(\operatorname{arctg} 2) + \operatorname{ctg}(\operatorname{arcctg} 7)} = \frac{\frac{2}{7} - 1}{2 + \frac{1}{7}} = -\frac{1}{3};$

**f)** $\frac{7\sqrt{130}}{130}$; **g)** $\frac{\sqrt{5} + \sqrt{2}}{2}.$

**24) a)** Se notează $a = \operatorname{arcctg} 3$ și $b = \frac{\pi}{4} - \arcsin\frac{1}{\sqrt{5}}$.

$\operatorname{tg} a = \frac{1}{3}$; $\operatorname{tg} b = \frac{1 - \operatorname{tg}\left(\arcsin\frac{1}{\sqrt{5}}\right)}{1 + \operatorname{tg}\left(\arcsin\frac{1}{\sqrt{5}}\right)} = \frac{1}{3}.$

$\operatorname{tg} a = \operatorname{tg} b$ și $a, b \in \left(0; \frac{\pi}{2}\right) \Rightarrow a = b.$

**b)** Se notează $a = 2\operatorname{arctg}\frac{1}{2}$ şi $b = \frac{\pi}{2} - \arccos\frac{4}{5}$.
$\sin a = \frac{2\operatorname{tg}\left(\operatorname{arctg}\frac{1}{2}\right)}{1+\operatorname{tg}^2\left(\operatorname{arctg}\frac{1}{2}\right)} = \frac{4}{5}$; $\sin b = \cos\left(\arccos\frac{4}{5}\right) = \frac{4}{5}$.
$\sin a = \sin b,\ a, b \in \left(0; \frac{\pi}{2}\right) \Rightarrow a = b$.
**c)** Egalitatea este echivalentă cu $\operatorname{arctg} 2 + \operatorname{arctg} 3 = \frac{3\pi}{4}$ etc.
**d)** $\cos\left(2\arccos\frac{3}{4}\right) = \cos\left(\arccos\frac{1}{8}\right)$ etc.
**e)** Se notează $a = \arcsin\frac{5}{13} + \arcsin\frac{16}{65}$ şi $b = \frac{\pi}{2} - \arcsin\frac{4}{5}$.
$\sin a = \sin b = \frac{3}{5},\ a, b \in \left(0; \frac{\pi}{2}\right) \Rightarrow a = b$.
**f)** $\cos\left(\arccos\frac{1}{7} + \arccos\frac{11}{14}\right) = -\frac{1}{2}$ etc.
**25) a)** $a = \sin\left(\arcsin\frac{\sqrt{3}}{3} + \arcsin\frac{2}{3}\right) = \frac{\sqrt{15}+2\sqrt{6}}{9}$.
**b)** $a = \cos\left(\arccos\frac{2}{3} - \arccos\frac{1}{3}\right) = \frac{2+2\sqrt{10}}{9}$.
**c)** $a = \operatorname{tg}(\operatorname{arctg} 2 + \operatorname{arctg} 5) = -\frac{7}{9}$.
**d)** $-7$; **e)** $\frac{2(9-5\sqrt{2})}{31}$.
**26) a)** $\arccos x,\ 2\arccos\sqrt{\frac{1+x}{2}} \in [0; \pi],\ \forall x \in [-1; 1]$.
$\cos(\arccos x) = \cos\left(2\arccos\sqrt{\frac{1+x}{2}}\right) = x$.
**b)** $\operatorname{tg}(\operatorname{arcctg} x) = \frac{1}{x},\ \forall x \in \mathbb{R}^*$.
$\operatorname{tg}\left(\arcsin\frac{1}{\sqrt{1+x^2}}\right) = \frac{\sin\left(\arcsin\frac{1}{\sqrt{1+x^2}}\right)}{\cos\left(\arcsin\frac{1}{\sqrt{1+x^2}}\right)} = \frac{1}{|x|},\ \forall x \in \mathbb{R}^*$.
Dacă $x = 0$, egalitatea are loc.
Dacă $x > 0$, atunci deoarece $\operatorname{tg}(\operatorname{arcctg} x) = \operatorname{tg}\left(\arcsin\frac{1}{\sqrt{1+x^2}}\right)$ şi $\operatorname{arcctg} x$, $\arcsin\frac{1}{\sqrt{1+x^2}} \in \left(0; \frac{\pi}{2}\right)$, rezultă $\operatorname{arcctg} x = \arcsin\frac{1}{\sqrt{1+x^2}}$.
Dacă $x < 0$, atunci deoarece $\operatorname{tg}(\operatorname{arcctg} x) = \frac{1}{x} = -\operatorname{tg}\left(\arcsin\frac{1}{\sqrt{1+x^2}}\right) =$ $= \operatorname{tg}\left(\pi - \arcsin\frac{1}{\sqrt{1+x^2}}\right)$ şi $\operatorname{arcctg} x,\ \pi - \arcsin\frac{1}{\sqrt{1+x^2}} \in \left(-\frac{\pi}{2};\ 0\right)$, rezultă $\operatorname{arcctg} x = \pi - \arcsin\frac{1}{\sqrt{1+x^2}}$.
**c)** Se procedează ca la subpunctul a).
**d)** Se procedează ca la subpunctul b).
**27)** Funcțiile de la **a)**, **d)** şi **g)** sunt impare; funcțiile de la **b)** şi **e)** sunt pare; funcțiile de la **c)** şi **f)** nu sunt nici pare, nici impare.
**28) a)** $[-\pi - 3; \pi - 3]$; **b)** $[2 - 3\pi; 2]$; **c)** $\left[\frac{\pi}{4}; \frac{\pi}{2}\right)$; **d)** $\left(2; 2 + \sqrt{\pi}\right)$; **e)** $\left[\sqrt{\frac{\pi}{6}}; \sqrt{\frac{2\pi}{3}}\right]$;
**f)** $\left[\frac{4}{3\pi}; \frac{2}{\pi}\right]$; **g)** $\left[0; \frac{\pi^2}{9}\right]$;
**29)** Se folosesc operațiile cu funcții monotone şi proprietățile legate de compunerea funcțiilor monotone.
**30) a)** $f$ este strict descrescătoare pe $\left[-\sqrt{2}; 0\right]$ şi strict crescătoare pe $\left[0; \sqrt{2}\right]$.
**b)** $f$ este strict crescătoare pe $(-\infty; -2]$ şi pe $[2; \infty)$.
**c)** $f$ este strict descrescătoare pe $(-\infty; 1]$ şi strict crescătoare pe $[1; \infty)$.

**d)** $f$ este strict descrescătoare pe $(-\infty; 0]$ şi strict crescătoare pe $[0; \infty)$.

**31) a)** $\frac{\sqrt{10}}{5} \in [0; 1] \Rightarrow \arcsin\frac{\sqrt{10}}{5} > 0$;

**b)** $\frac{\sqrt{3}-\sqrt{5}}{4} \in [-1; 0] \Rightarrow \arcsin\frac{\sqrt{3}-\sqrt{5}}{4} < 0$;

**c)** $\arccos\left(-\frac{1}{3}\right) > 0$; **d)** $\arccos\frac{2}{7} > 0$; **e)** $\operatorname{arctg}\left(\sqrt{7}-\sqrt{3}\right) > 0$; **f)** $\operatorname{arctg} 8 > 0$;

**g)** $\operatorname{arcctg}(-3) > 0$; **h)** $\operatorname{arcctg}\sqrt{5} > 0$; **i)** $\arcsin\frac{1}{2} - \arcsin\frac{1}{3} > 0$;

**j)** $\arccos\frac{\sqrt{3}}{4} - \arccos\frac{1}{3} < 0$; **k)** $\operatorname{arctg}\frac{1}{8} - \operatorname{arctg} 2 < 0$; **l)** $\operatorname{arcctg} 2009 - \pi < 0$;

**m)** $\frac{\pi}{4} - \operatorname{arctg} 2 < 0$; **n)** $\frac{\pi}{4} - \arccos\frac{1}{3} < 0$.

**32) a)** $f(x) = \begin{cases} -\arcsin x - \frac{\pi}{6}, x \in \left[-1; -\frac{1}{2}\right] \\ \arcsin x + \frac{\pi}{6}, x \in \left(-\frac{1}{2}; 1\right] \end{cases}$;

**b)** $f(x) = \begin{cases} \arccos x - \frac{\pi}{2}, x \in [-1; 0] \\ \frac{\pi}{2} - \arccos x, x \in (0; 1] \end{cases}$;

**c)** $f(x) = \begin{cases} \frac{\pi}{4} - \operatorname{arctg} x, x \in (-\infty; 1] \\ \operatorname{arctg} x - \frac{\pi}{4}, x \in (1; \infty) \end{cases}$;

**d)** $f(x) = \begin{cases} \operatorname{arcctg} x - \frac{3\pi}{4}, x \in (-\infty; -1] \\ \frac{3\pi}{4} - \operatorname{arcctg} x, x \in (-1; \infty) \end{cases}$;

**e)** $f(x) = \begin{cases} -2, x \in [-1; -\sin 1) \\ -1, x \in [-\sin 1; 0) \\ 0, x \in [0; \sin 1) \\ 1, x \in [\sin 1; 1] \end{cases}$;

**f)** $f(x) = \begin{cases} \arccos x - 3, x \in [-1; \cos 3] \\ \arccos x - 2, x \in (\cos 3; \cos 2] \\ \arccos x - 1, x \in (\cos 2; \cos 1] \\ \arccos x \quad, x \in (\cos 1; 1] \end{cases}$;

**g)** $f(x) = \begin{cases} \operatorname{arcctg} x, x \in (-\infty; 1] \\ \operatorname{arctg} x, x \in (1; \infty) \end{cases}$;

**h)** $f(x) = \begin{cases} \arcsin x, x \in \left[-1; \frac{\sqrt{2}}{2}\right] \\ \arccos x, x \in \left(\frac{\sqrt{2}}{2}; 1\right] \end{cases}$.

**33) a)** $f$ este inversabilă $\Leftrightarrow \forall y \in [-1; 1], \exists!\, x \in \left[\frac{\pi}{2}; \frac{3\pi}{2}\right]$ astfel încât $f(x) = y$.

Ecuația $\sin x = y, x \in \mathbb{R}, y \in [-1; 1]$ are soluțiile $x_k = (-1)^k \arcsin y + k\pi, k \in \mathbb{Z}$.

$x_1 = \pi - \arcsin y$ este unica soluție din intervalul $\left[\frac{\pi}{2}; \frac{3\pi}{2}\right]$ (funcția $f$ este strict crescătoare).

$f^{-1}: [-1; 1] \to \left[\frac{\pi}{2}; \frac{3\pi}{2}\right], f^{-1}(x) = \pi - \arcsin x$.

**b)** $f^{-1}: \mathbb{R} \to \left(-\frac{3\pi}{2}; -\frac{\pi}{2}\right), f^{-1}(x) = \operatorname{arctg} x - \pi$.

**c)** $f^{-1}: [-1; 1] \to \left[-\frac{\pi}{2}; 0\right], f^{-1}(x) = -\frac{1}{2}\arccos x$.

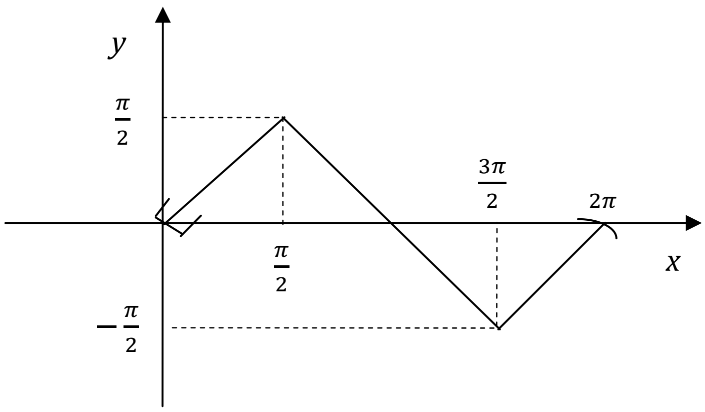
**d)** $f^{-1}: (0; 2\pi) \to \mathbb{R}, f^{-1}(x) = \operatorname{ctg}\frac{x}{2}$.

**e)** $f^{-1}: \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \to [0; 1], f^{-1}(x) = \frac{1+\sin x}{2}$.

**f)** $f^{-1}: [-3\pi; -\pi] \to [-1; 1], f^{-1}(x) = \cos\frac{3\pi+x}{2}$.
**g)** $f^{-1}: \left(-\frac{\pi}{2}; \frac{\pi}{2}\right) \setminus \left\{\frac{\pi}{4}\right\} \to \mathbb{R} \setminus \{1\}, f^{-1}(x) = \frac{\operatorname{tg} x+1}{\operatorname{tg} x-1}$.
**h)** $f^{-1}: (0; \pi) \to \mathbb{R}, f^{-1}(x) = \operatorname{ctg}\frac{x^2}{\pi}$.
**34) a)** $f(x + 2\pi) = f(x), \forall x \in \mathbb{R}$;

**b)** $f(x) = \begin{cases} x, & x \in \left[0; \frac{\pi}{2}\right) \\ \pi - x, & x \in \left[\frac{\pi}{2}; \frac{3\pi}{2}\right] \\ x - 2\pi, & x \in \left(\frac{3\pi}{2}; \pi\right) \end{cases}$;

**c)**




**d)** $\frac{\pi}{4}; \frac{3\pi}{4}$; **e)** $x \in \left[0; \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}; 2\pi\right)$.
**35) a)** $f(-x) = f(x), \forall x \in \mathbb{R}$.
**b)** Se consideră funcția $g_1 : \mathbb{R} \to [-1; 1], g_1(x) = \frac{x^2-1}{x^2+1}$,
$a, b \in \mathbb{R}, a \neq b \Rightarrow \frac{g_1(a)-g_1(b)}{a-b} = 2(a + b)$.
$g_1$ este strict descrescătoare pe $(-\infty; 0]$ şi strict crescătoare pe $[0; \infty)$.
$f = h \circ g_1$, unde $h : [-1; 1] \to \mathbb{R}, h(x) = \arcsin x$ este strict crescătoare.
Funcția $f$ are aceleaşi intervale de monotonie ca şi funcția $g_1$.
**c)** Ecuația $\arcsin\frac{x^2-1}{x^2+1} = y,\ y \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right)$ are în $[0; \infty)$ soluția unică $x = \sqrt{\frac{1+\sin y}{1-\sin x}}$, $\forall y \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right)$.
$g^{-1} : \left[-\frac{\pi}{2}; \frac{\pi}{2}\right) \to [0; \infty), g^{-1}(x) = \sqrt{\frac{1+\sin x}{1-\sin x}}$.
**36) a)** $f(1) = f(-1)$.
**b)** $f = h_2 \circ h_1$, unde $h_1 : [0; \infty) \to (-1; 1], h_1(x) = \frac{1-x^2}{1+x^2}$ şi $h_2 : (-1; 1] \to \mathbb{R}$, $h_2(x) = \operatorname{arctg} x$.
$a, b \in [0; \infty), a \neq b \Rightarrow \frac{h_1(a)-h_1(b)}{a-b} = -2(a + b) \leq 0$.
$h_1$ strict descrescătoare şi $h_2$ strict crescătoare $\Rightarrow h = h_2 \circ h_1$ strict descrescătoare.
**c)** $\mathrm{A} = \operatorname{Im} h_1 = \left(-\frac{\pi}{4}; \frac{\pi}{4}\right]$.
$g^{-1} : \left[-\frac{\pi}{4}; \frac{\pi}{4}\right) \to [0; \infty), g^{-1}(y) = \sqrt{\frac{1-\operatorname{tg} y}{1+\operatorname{tg} y}}$.
**37) a)** Presupunem că există $\mathrm{T} > 0$ astfel încât $f(x + \mathrm{T}) = f(x), \forall x \in \mathbb{R}$, deci
$\mathrm{T} + \sin(x + \mathrm{T}) = \sin x, \forall x \in \mathbb{R}$.

Dacă $x = 0$, rezultă $\sin \mathrm{T} = -\mathrm{T}$, deci $\mathrm{T} \in (0; 1] \subset \left(0; \frac{\pi}{2}\right)$; atunci $\sin \mathrm{T} > 0$ și egalitatea $\sin \mathrm{T} = -\mathrm{T}$ este imposibilă.

**b)** Se procedează ca la subpunctul a).

**c)** Presupunem că există $\mathrm{T} > 0$ astfel încât $\sin(x^2) = \sin(x + \mathrm{T})^2$, $\forall x \in \mathbb{R}$.

Dacă $x = 0$, atunci $\sin \mathrm{T}^2 = 0$, deci $\mathrm{T}_n = \sqrt{n\pi}$, $n \in \mathbb{N}^*$.

$\sin(x^2) = \sin\left(x + \sqrt{n\pi}\right)^2$, $\forall x \in \mathbb{R}$, $\forall n \in \mathbb{N}^* \Rightarrow \sin(x^2) = \sin\left(x^2 + 2x\sqrt{n\pi} + n\pi\right)$, $\forall x \in \mathbb{R}$, $\forall n \in \mathbb{N}^*$.

Dacă $n$ este par, avem

$\sin(x^2) = \sin\left(x^2 + 2x\sqrt{n\pi}\right)$, $\forall x \in \mathbb{R}$, de unde $x^2 = x^2 + 2x\sqrt{n\pi} + 2k\pi$, $\forall x \in \mathbb{R}$, $\forall k \in \mathbb{Z}$.

Prin ridicare la pătrat se obține $x^2 n = k\pi$, $\forall x \in \mathbb{R}$, $\forall k \in \mathbb{Z}$.

Ultima egalitate nu are loc dacă $x \in \mathbb{Q}^*$.

**38) a)** $|\sin x + \cos x| = \sqrt{2}\left|\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}\right| = \sqrt{2}\left|\sin\left(x + \frac{\pi}{4}\right)\right| \leq \sqrt{2}$, $\forall x \in \mathbb{R}$.

**b)** Inegalitatea este echivalentă cu $\sqrt{2}\left|\sin\left(x - \frac{\pi}{4}\right)\right| \leq \sqrt{2}$, $\forall x \in \mathbb{R}$.

**c)** Inegalitatea este echivalentă cu $2\left|\sin\left(x + \frac{\pi}{6}\right)\right| \leq 2$, $\forall x \in \mathbb{R}$.

**d)** $2\left|\sin\left(x - \frac{\pi}{6}\right)\right| \leq 2$, $\forall x \in \mathbb{R}$.

**39) a)** $(1 + \cos x)(2\sin x + 2) + 1 \geq 0$, $\forall x \in \mathbb{R}$.

**b)** $\sin x \cdot \sin 2x \cdot \sin 3x = \frac{1}{2}(\cos x - \cos 3x)\sin 3x = \frac{1}{4}(\sin 4x + \sin 2x - \sin 6x)$.

$\sin 4x \leq 1$, $\forall x \in \mathbb{R}$; $\sin 2x \leq 1$, $\forall x \in \mathbb{R}$; $-\sin 6x \leq 1$, $\forall x \in \mathbb{R}$. În aceste inegalități egalitățile nu au loc simultan.

**c)** $2\cos 2x + 2\cos x + 3 \geq 0, \forall x \in \mathbb{R} \Leftrightarrow 4\cos^2 x + 2\cos x + 1 \geq 0, \forall x \in \mathbb{R} \Leftrightarrow$

$\Leftrightarrow \left(2\cos x + \frac{1}{2}\right)^2 + \frac{3}{4} \geq 0, \forall x \in \mathbb{R}$.

**d)** $\cos x < \cos\frac{x}{3}\cos\frac{2x}{3}, \forall x \in (0;\pi) \Leftrightarrow 4\cos^3\frac{x}{3} - 3\cos\frac{x}{3} < \cos\frac{x}{3}\left(2\cos^2\frac{x}{3} - 1\right)$,

$\forall x \in (0;\pi) \Leftrightarrow \cos\frac{x}{3}\left(2\cos^2\frac{x}{3} - 2\right) < 0, \forall x \in (0;\pi)$

Ultima inegalitate este adevărată deoarece $\cos\frac{x}{3} > 0$, $\forall x \in (0;\pi)$ și $\cos^2\frac{x}{3} < 1$, $\forall x \in (0;\pi)$.

**e)** $2(\sin^4 x + \cos^4 x) \geq 1 \Leftrightarrow 2[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x] \geq 1 \Leftrightarrow$

$\Leftrightarrow 2\left(1 - \frac{1}{2}\sin^2 2x\right) \geq 1 \Leftrightarrow \sin^2 2x \leq 1$.

**f)** Inegalitatea este echivalentă cu $\frac{\sin x(2\cos^2 x - \cos x - 1)}{\cos x} < 0, \forall x \in \left(0; \frac{\pi}{2}\right)$, apoi cu

$2\cos^2 x - \cos x - 1 < 0, \forall x \in \left(0; \frac{\pi}{2}\right)$ sau cu $\cos x \in \left(-\frac{1}{2}; 1\right), \forall x \in \left(0; \frac{\pi}{2}\right)$ (adevărat).

**g)** Dacă $\sin a = 1$, inegalitatea devine $2 \geq 0$ (adevărat).

Dacă $\sin a \neq 1$, inegalitatea este adevărată deoarece $1 - \sin a > 0$ și $\Delta = 0$.

**1) a)** 2; **b)** 2; **c)** 1; **d)** 0… ; **m)** 3; **n)** 6.

**2) a)** 4; **b)** 5; **c)** 2; **d)** 0…

**3)** $A_1, A_2, A_3, A_4, A_9, A_1$…

**4) a)** $[0; 1]$; **b)** $(-\infty; -$… $^*$; **e)** $\mathbb{R} \setminus \{3\}$; **f)** $[-1; 2]$;

**g)** $[-\sqrt{5}; -\sqrt{3}] \cup [\sqrt{3};$ …